6.3/40

$$\int \sin(\sin\theta)\cos\theta\, d\theta$$

Let $u = \sin\theta$

$du = \cos\theta\, d\theta$

$$\int \sin(u)\, du$$

$$-\cos(\sin\theta) + C \Leftarrow -\cos u + C$$

47)

$$\int \sin^3(2\theta)\,d\theta$$

$\sin^2+\cos^2=1$

$\sin^2=1-\cos^2$

$$= -\frac{1}{2}\int -2\sin(2\theta)\left(1-\cos^2(2\theta)\right)d\theta$$

Let $u=\cos(2\theta)$

$du=-2\sin(2\theta)\,d\theta$

$$-\frac{1}{2}\int 1-u^2\,du$$

$$= -\frac{1}{2}\left(u-\frac{u^3}{3}\right)+C \ldots$$

42)
$$\int \sqrt{e^x}\,dx = \int (e^x)^{1/2}\,dx$$

$$= \int e^{\frac{x}{2}}\,dx \Rightarrow 2\int e^u\,du$$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2du = dx$

$$= 2e^u + C = 2e^{\frac{x}{2}} + C$$

48)

$$\int \sec^4(3\theta)\, d\theta$$

$$\sin^2 + \cos^2 = 1$$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\tan^2 + 1 = \sec^2$$

$$\frac{1}{3}\int 3\sec^2(3\theta)\left[\tan^2(3\theta)+1\right] d\theta$$

$$\int (\tan^2+1)^2\, d\theta$$

$$\int \tan^4 + 2\tan^2 + 1 \ldots$$

$$u = \tan(3\theta)$$

$$du = 3\sec^2(3\theta)\, d\theta$$

$$\frac{1}{3}\int u^2+1 \; du$$

44)

$$\frac{1}{2}\int \frac{dy}{2\sqrt{y}\,e^{\sqrt{y}}}$$

Let $u=\sqrt{y}$

$$du = \frac{1}{2\sqrt{y}}\,dy$$

$$\Rightarrow \frac{1}{2}\int \frac{1}{e^{u}}\,du$$

$$= \frac{1}{2}\int e^{-u}\,du$$

Let $w=-u$

$dw=-du$

$$\frac{1}{2}\int e^{w}\,dw$$

$$\frac{1}{2}e^{w}+C$$

$$\frac{1}{2}e^{-u}+C$$

$$\frac{1}{2}e^{-\sqrt{y}}+C$$

$$u=\sqrt{y}=y^{1/2}$$

$$\frac{du}{dy}=\frac{1}{2}y^{-1/2}$$

$$du=\frac{1}{2}y^{-\frac{1}{2}}dy=\frac{1}{2\sqrt{y}}dy$$

4b

$$\int \frac{y}{\sqrt{y+1}}\,dy \;\Rightarrow\; \int \frac{(u-1)}{\sqrt{u}}\,du$$

Let $u = y+1$

$du = dy$

$y = u-1$

$$= \int \frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\,du$$

$$= \int \sqrt{u} - \frac{1}{\sqrt{u}}\,du$$

$$= \int u^{1/2} - u^{-1/2}\,du$$

$$= \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} + C$$

$$= \frac{(y+1)^{3/2}}{3/2} - \frac{(y+1)^{1/2}}{1/2} + C$$

6.4 § Sigma Notation

$$\prod_{k=1}^{5} k^2 = 1^2\cdot 2^2\cdot 3^2\cdot 4^2\cdot 5^2$$

$$\sum_{k=1}^{5} k^2 = 1^2+2^2+3^2+4^2+5^2$$

$$= \underline{55}$$

Σ: add up the terms I get when I replace the index variable with initial number ~~and~~ successors ($+1$)

5: I will stop after index variable is replaced by this number

k^2: my pattern — think "function"

k: the variable that will change in my pattern — aka the index variable

1: where k begins

$$A(x,y) = x+y$$
$$S(x,y) = x-y$$
$$M(x,y) = x\cdot y$$

$$\sum_{j=1}^{4} \sum_{k=1}^{5} a_{jk} = \sum_{j=1}^{4} \left(a_{j1} + a_{j2} + a_{j3} + a_{j4} + a_{j5} \right)$$

$$a_{11} + a_{12} + a_{13} + a_{14} + a_{15}$$

$$+ a_{21} + a_{22} + \cdots$$

Series

$$\sum_{k=1}^{5}(2k) = 2+4+6+8+10$$

$$\sum_{k=0}^{4}(2k+2) = \underset{2(0)+2}{2}+\underset{2(1)+2}{4}+\underset{2(2)+2}{6}+\underset{2(3)+2}{8}+\underset{2(4)+2}{10}$$

$$\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+4^2+\dots+n^2$$

Partial Sum

n=	1	2	3	4	5	
	1,	5,	14,	30,	55,	...
D1:		4	9	16	25	...
D2:			5	7	9	11 ...
D3:				2	2	2

6

$$\frac{2n^3}{6}$$

n=	1	2	3	4	5
	1	5	14	30	55
−	$\frac{2}{6}$	$\frac{16}{6}$	$\frac{54}{6}$	$\frac{128}{6}$	$\frac{250}{6}$
new seq:	$\frac{4}{6}$,	$\frac{14}{6}$,	$\frac{30}{6}$,	$\frac{52}{6}$,	$\frac{80}{6}$
D1		$\frac{10}{6}$	$\frac{16}{6}$	$\frac{22}{6}$,	$\frac{28}{6}$
D2			$\frac{6}{6}=1$	$\frac{6}{6}=1$	$\frac{6}{6}=1$

So pattern formula is now

$$\frac{2n^3}{6}+\frac{1n^2}{2}+\dots$$

6.4/ 1-8

6.3/49

$$\int \frac{t+1}{t}\,dt = \int \frac{t}{t} + \frac{1}{t}\,dt$$

$$= \int 1 + t^{-1}\,dt$$

$$= t + \ln t + C$$

0^0 ?

consider

$\lim_{x\to 0} x^0 = 1$

consider

$\lim_{x\to 0} 0^x = 0$ (from right side) und (from left side)

Consider $x^x = f(x)$

what is $f(-2)$? $\frac{1}{4}$

$f(-3)$? $\frac{1}{-27}$

$f(-\frac{1}{2}) = (-\frac{1}{2})^{-\frac{1}{2}} = \frac{1}{\sqrt{-\frac{1}{2}}}$ und

6.3/45 $\int x\sqrt{x-3}\,dx$

$u = x-3$

$du = dx$

$u+3 = x$

$\int (u+3)\sqrt{u}\,du$

$= \int u^{3/2} + 3u^{1/2}\,du$

$\vdots$

6.3/47 $\int \sin^3 2\theta\,d\theta = -\frac{1}{2}\int -2\sin(2\theta)\left(1-\cos^2(2\theta)\right)d\theta$

$u = \cos(2\theta)$

$du = -2\sin(2\theta)d\theta$

$-\frac{1}{2}\int 1-u^2\,du = \dots$

48 $\int \sec^4(3\theta)\,d\theta$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\tan^2 + 1 = \sec^2$$

$$= \frac{1}{3}\int \boxed{3\sec^2(3\theta)}\,(1+\tan^2(3\theta))\,\boxed{d\theta}$$

$u = \tan(3\theta)$

$du = \boxed{3\sec^2(3\theta)\,d\theta}$

$$\frac{1}{3}\int 1+u^2\,du \ldots$$

49 $$\int \frac{t+1}{t}\,dt = \int \frac{t}{t} + \frac{1}{t}\,dt$$

$$= t + \ln|t| + C$$

6.4 Sigma Notation

$$\prod_{k=1}^{5} k^2 = 1^2 \cdot 2^2 \cdot 3^2 \cdot 4^2 \cdot 5^2$$

add up all the terms, replacing the k with the initial number (1 here) and each successor, stopping at first number

stop when I've replaced the k with this number

$$\sum_{k=1}^{5} k^2$$

pattern; think "function"

$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= 55$

the variable that changes (by +1 each iteration) in the pattern. AKA the index variable.

start k with this number

$$\sum_{j=1}^{2}\sum_{k=1}^{5} a_{jk} = \cancel{a_{j1} + a_{j2} + a_{j3} + a_{j4}} + a_{j5}$$

$$a_{11} + a_{12} + a_{13} + a_{14} + a_{15}$$

$$+ a_{21} + a_{22} + a_{23} + a_{24} + a_{25}$$

Eggsample

a_{11} a_{12} a_{13} $\cdot$

a_{21} a_{22} $\cdot$ $\cdot$ $\cdot$

$$\begin{bmatrix} 1 & 3 & 7 & -2 & 0 \\ 5 & 0 & 15 & 6 & 2 \end{bmatrix}$$

add
up
terms
:
series

$$\sum_{k=1}^{5} 2k = 2+4+6+8+10 = 30$$

$$\sum_{k=0}^{4} 2k+2 = 2+4+6+8+10$$

$$\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+4^2+\dots+n^2$$

$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
1	5	14	30	55	91

D1: 4 9 16 25 36

D2: 5 7 9 11

D3: 2 2 2

$$\frac{2n^3}{6} = \frac{n^3}{3}$$

	$n=1$	$n=2$	$n=3$	$n=4$		
	1	5	14	30	55	91
$\frac{n^3}{3}$:	$-\frac{1}{3}$	$-\frac{8}{3}$	$-\frac{27}{3}$	$-\frac{64}{3}$	$-\frac{125}{3}$	$-\frac{216}{3}$
New Pattern	$\frac{2}{3}$	$\frac{7}{3}$	$\frac{15}{3}$	$\frac{26}{3}$	$\frac{40}{3}$	$\frac{57}{3}$

D1: $\frac{5}{3}$ $\frac{8}{3}$ $\frac{11}{3}$ $\frac{14}{3}$ $\frac{17}{3}$

D2: 1 1 1 1

$\frac{n^2}{2}$

	$\frac{2}{3}$	$\frac{7}{3}$	$\frac{15}{3}$	$\frac{26}{3}$	$\frac{40}{3}$	$\frac{57}{3}$
$\frac{n^2}{2}$	$-\frac{1}{2}$	$-\frac{4}{2}$	$-\frac{9}{2}$	$-\frac{16}{2}$	$-\frac{25}{2}$	$-\frac{36}{2}$
	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$

D1: $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

$\frac{n}{6}$

$$\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6} = \boxed{\text{sum of } n \text{ squares}}$$

Add up first 69 squares

$$\frac{69^3}{3}+\frac{69^2}{2}+\frac{69}{6} = 111\ 895$$